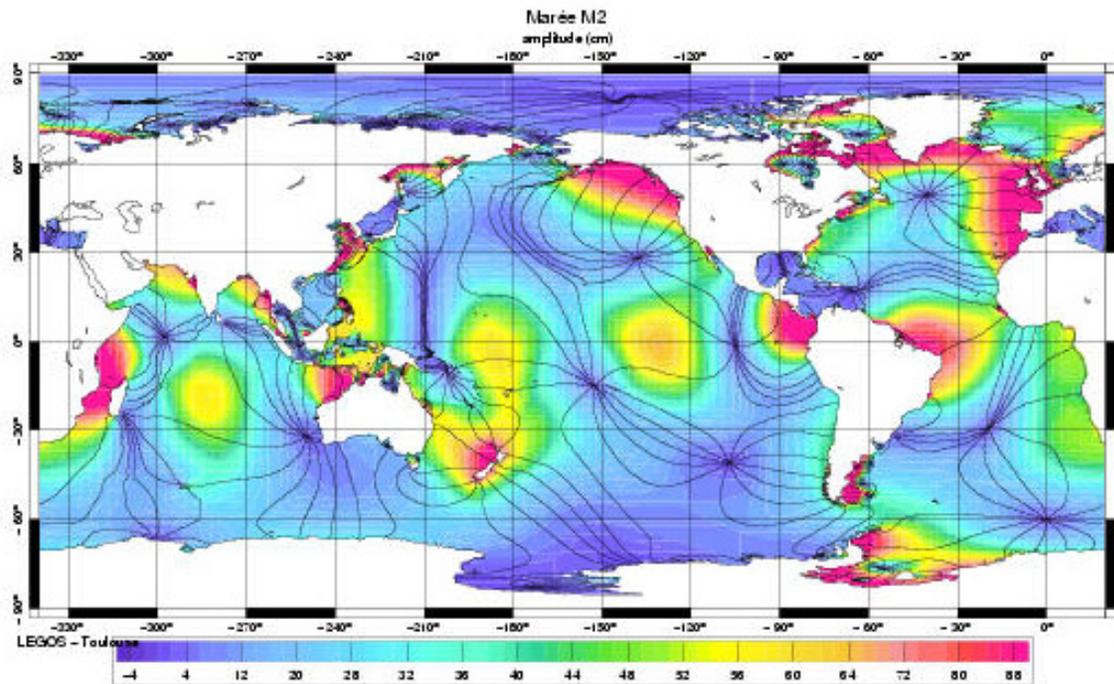


# *Ocean and atmospheric tides standards (used for EIGEN gravity field modeling)*

*Richard Biancale (CNES/GRGS)*



# Ocean tides modeling

The height of ocean tides is expressed by a sum over  $n$  waves :

$$\xi(\varphi, \lambda, t) = \sum_n Z_n(\varphi, \lambda) \cos(\theta_n(t) - \psi_n(\varphi, \lambda))$$

$Z_n$  is the amplitude of the wave  $n$ ,

$\psi_n$  is the phase

$\theta_n$  is the Doodson argument which is expressed in linear combination of 6 variables :

$$\theta_n(t) = n_1 \tau + (n_2 - 5)s + (n_3 - 5)h + (n_4 - 5)p + (n_5 - 5)N' + (n_6 - 5)p_s$$

These 6 variables with decreasing frequencies represent the fundamental arguments according to the Sun and Moon motions :

$\tau$  : angle of the mean lunar day (1.03505 d)

$s$  : angle of the mean tropic month (27.32158 d)

$h$  : angle of the mean tropic year (365.2422 d)

$p$  : angle of the mean lunar perigee (8.8473 y)

$N'$  : angle of the mean lunar node (18.6129 y)

$p_s$  : angle of the perihelion (20940.28 y)

$n_1$  (= 0, 1, 2, 3...) defines the specie (long period, diurnal, semi-diurnal, ter-diurnal...),  $n_2$  the group (in general :  $1 \leq n_2 \leq 9$ ) and  $n_3$  the constituent ( $1 \leq n_3 \leq 9$ ).

The amplitude ( $Z_n$ ) and phase ( $\psi_n$ ) of the different waves of tides represented by **cotidal maps** can be expanded in spherical harmonic functions of  $Z_n \cos \psi_n$  and  $Z_n \sin \psi_n$  :

$$\begin{cases} Z_n \cos \psi_n = \sum_l \sum_m (a_{n,lm} \cos m\lambda + b_{n,lm} \sin m\lambda) P_{lm}(\sin \varphi) \\ Z_n \sin \psi_n = \sum_l \sum_m (c_{n,lm} \cos m\lambda + d_{n,lm} \sin m\lambda) P_{lm}(\sin \varphi) \end{cases}$$

Then we have :  $\xi(\varphi, \lambda, t) = \sum_n (Z_n \cos \psi_n \cos \theta_n + Z_n \sin \psi_n \sin \theta_n)$

$$= \sum_n \sum_l \sum_m \left[ \begin{aligned} & \frac{a_{n,lm} - d_{n,lm}}{2} (\cos m\lambda \cos \theta_n - \sin m\lambda \sin \theta_n) \\ & + \frac{a_{n,lm} + d_{n,lm}}{2} (\cos m\lambda \cos \theta_n + \sin m\lambda \sin \theta_n) \\ & + \frac{c_{n,lm} + b_{n,lm}}{2} (\cos m\lambda \sin \theta_n + \sin m\lambda \cos \theta_n) \\ & + \frac{c_{n,lm} - b_{n,lm}}{2} (\cos m\lambda \sin \theta_n - \sin m\lambda \cos \theta_n) \end{aligned} \right] P_{lm}(\sin \varphi)$$

Writing :

$$C_{n,lm}^+ = \frac{a_{n,lm} - d_{n,lm}}{2} , \quad C_{n,lm}^- = \frac{a_{n,lm} + d_{n,lm}}{2} \quad \text{and} \quad S_{n,lm}^+ = \frac{c_{n,lm} + b_{n,lm}}{2} , \quad S_{n,lm}^- = \frac{c_{n,lm} - b_{n,lm}}{2} ,$$

the **height of tide** becomes :  $\xi(\varphi, \lambda, t) = \sum_n \sum_l \sum_m P_{lm}(\sin \varphi) \sum_{\pm} [C_{n,lm}^{\pm} \cos(\theta_n + \chi_n \pm m\lambda) + S_{n,lm}^{\pm} \sin(\theta_n + \chi_n \pm m\lambda)]$

## Procedure

# reading the point grid by *lec\_grille(\_quart)\_degre*

# converting by *gpgm2*; point grid C  $\rightarrow$  mean grid C

# converting by *analhs*; mean grid C  $\rightarrow$   $a_{lm}, b_{lm}$  harmonics

# converting by *gpgm2*; point grid S  $\rightarrow$  mean grid S

# converting by *analhs*; mean grid S  $\rightarrow$   $c_{lm}, d_{lm}$  harmonics

# converting by *convers\_hs*;  $(a_{lm}, d_{lm}) / (b_{lm}, c_{lm})$  harmonics  $\rightarrow$   $C_{lm}^{\pm} / S_{lm}^{\pm}$  ocean tides harmonics

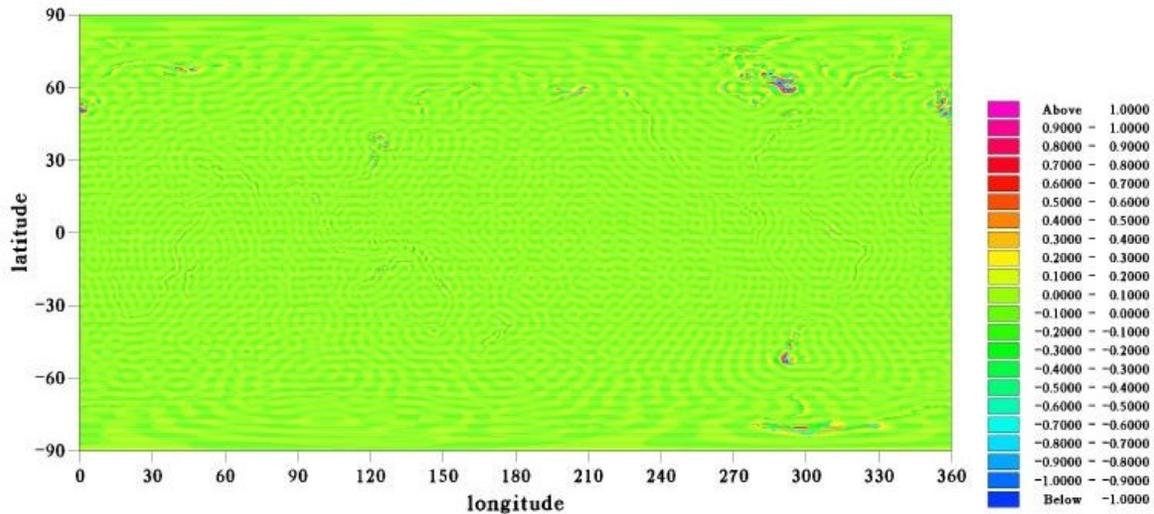
# applying by *cor\_ellips* the ellipsoidal correction

M2 - FES2004 model

$h * \cos(\text{phase})$  in metre

discrepancies

(rms : 0.0537 / moy : 0.0000 / min : -2.4076 / max : 2.8301)

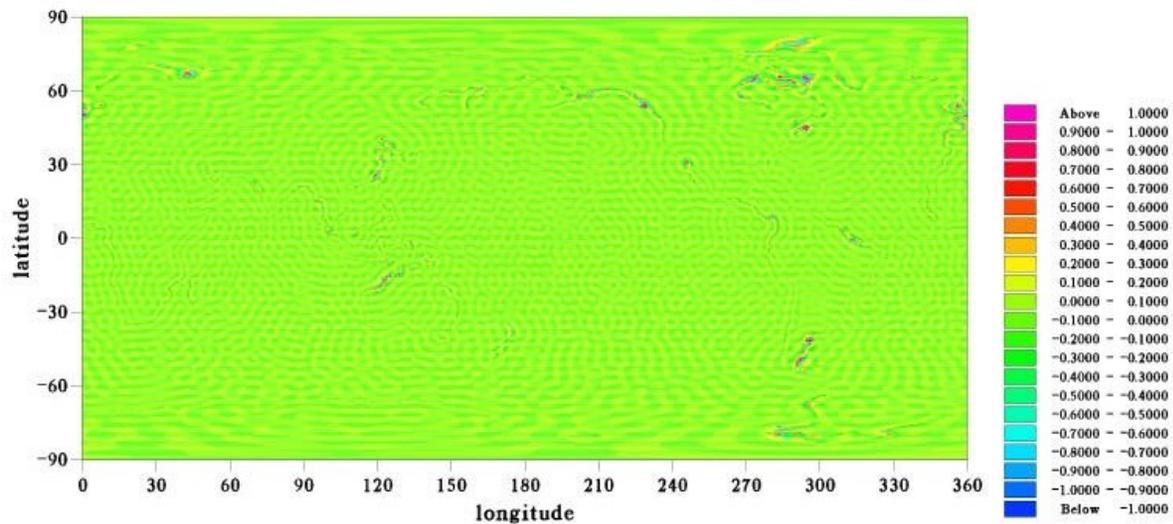


M2 - FES2004 model

$h * \sin(\text{phase})$  in metre

discrepancies

(rms : 0.0555 / moy : 0.0000 / min : -3.3586 / max : 1.8381)



# FES2004

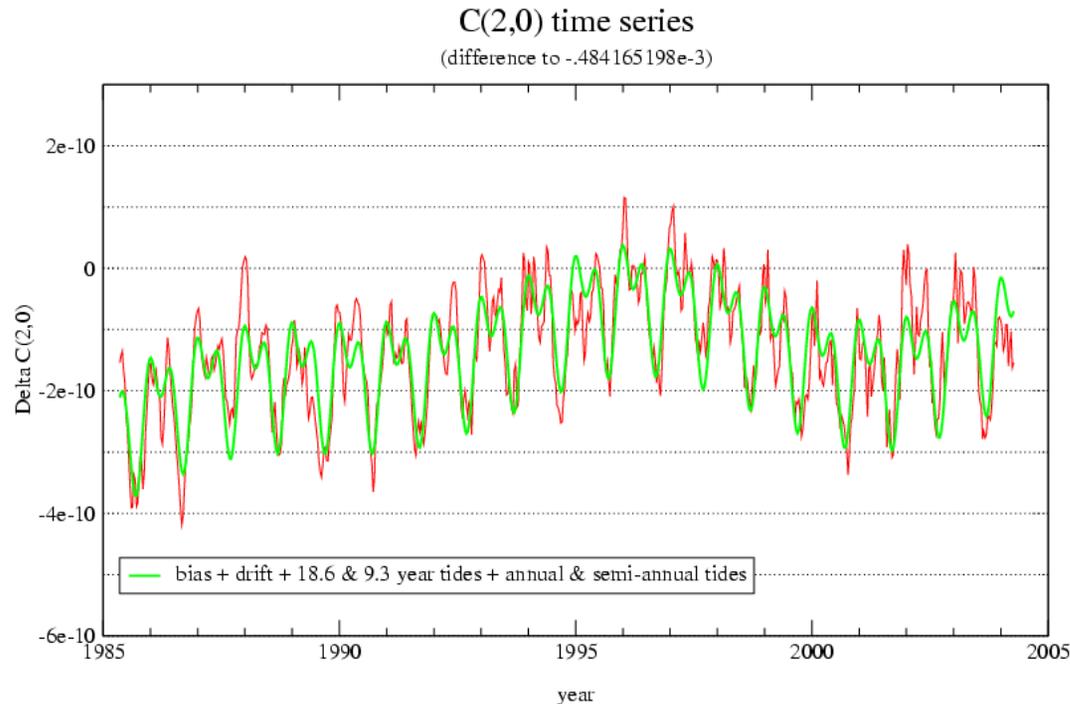
**Ocean tide model: FES2004 normalized model (fev. 2004) up to (100,100) in cm  
(long period from FES2002 up to (50,50) + equilibrium Om1/Om2, atmospheric tide NOT included)**

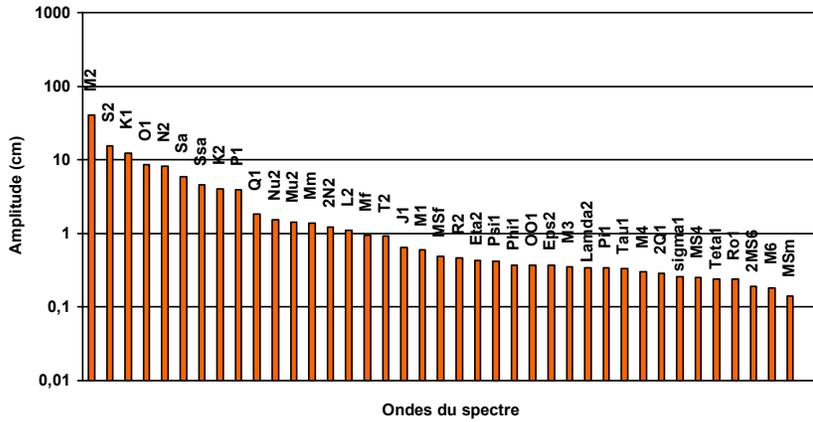
Doodson	l	m	Csin+	Ccos+	Csin-	Ccos-	C+	eps+	C-	eps-
55.565 Om1	2	0	-0.540594	0.000000	0.000000	0.000000	0.5406	270.000	0.0000	0.000
55.575 Om2	2	0	-0.005218	0.000000	0.000000	0.000000	0.0052	270.000	0.0000	0.000
56.554 Sa	2	0	-0.046604	-0.000903	0.000000	0.000000	0.0466	268.890	0.0000	0.000
57.555 Ssa	2	0	-0.296385	-0.010794	0.000000	0.000000	0.2966	267.914	0.0000	0.000
65.455 Mm	2	0	-0.479140	-0.084083	0.000000	0.000000	0.4865	260.047	0.0000	0.000
75.555 Mf	2	0	-0.805539	-0.236132	0.000000	0.000000	0.8394	253.662	0.0000	0.000
85.455 Mtm	2	0	-0.139082	-0.049418	0.000000	0.000000	0.1476	250.439	0.0000	0.000
93.555 Msq	2	0	-0.019391	-0.006674	0.000000	0.000000	0.0205	251.008	0.0000	0.000
135.655 Q1	2	1	-0.291290	0.308134	0.049563	-0.203915	0.4240	316.610	0.2099	166.339
145.555 O1	2	1	-1.480706	1.383072	0.455121	-0.792480	2.0262	313.047	0.9139	150.131
163.555 P1	2	1	-0.505544	0.551083	0.279251	-0.259244	0.7478	317.468	0.3810	132.872
165.555 K1	2	1	-1.530097	1.660923	0.845110	-0.785011	2.2583	317.348	1.1535	132.889
235.755 2N2	2	2	-0.044241	0.134832	0.020987	-0.001671	0.1419	341.834	0.0211	94.552
245.655 N2	2	2	-0.577967	0.958489	0.162389	0.009450	1.1193	328.910	0.1627	86.670
255.555 M2	2	2	-3.233275	3.840723	0.786314	0.430793	5.0205	319.908	0.8966	61.283
273.555 S2	2	2	-1.288610	1.293664	0.040901	0.419782	1.8259	315.112	0.4218	5.565
275.555 K2	2	2	-0.337696	0.358821	-0.000662	0.114679	0.4927	316.737	0.1147	359.669
455.555 M4	4	4	-0.000551	-0.003144	0.001155	-0.002202	0.0032	189.949	0.0025	152.313

# LAGEOS ocean tides fit over 19 y (normalized coefficients) :

		LAGEOS		FES-2004		difference			
doodson	darw	1	m	$C^+(\text{cm})$	$\varepsilon^+(\text{deg})$	$C^+(\text{cm})$	$\varepsilon^+(\text{deg})$	$\Delta C^+(\%)$	$\Delta \varepsilon^+(\%)$
<b>55.565</b>	<b><math>\Omega_1</math></b>	<b>2</b>	<b>0</b>	<b>0.4387</b>	<b>223.72</b>	<b>0.5406</b>	<b>270.00</b>	<b>19.</b>	<b>26.</b>
55.575	$\Omega_2$	2	0	0.3206	125.05	0.0052	270.00	6044.	80.
56.554	Sa	2	0	0.5800	26.61	0.0466	268.89	1144.	65.
57.555	Ssa	2	0	0.7390	277.17	0.2966	267.91	149.	5.

*$\Omega_2$ , Sa and Ssa results are not to be considered in terms of tides but more probably in terms of mass displacement*





Décomposition harmonique du spectre de marée (pour la base de données pélagiques ST95)

Onde	Pourcentage d'importance
$M_2$	33,5%
$S_2$	12,6%
$K_1$	10,1%
$O_1$	7,0%
$N_2$	6,8%
$K_2$	3,3%
$Q_1$	1,5%
$2N_2$	1,0%
Total	75,9%

Pourcentage d'importance des principales ondes du spectre (source : thèse F. Lefèvre)

Nom de Darwin	Nombre de Doodson	Argument de Doodson	Coef. harm. $\alpha_i$	Fréquence ( $^\circ/h$ )	Fréquence (rad/s)	Période (jours)	Origine
$M_0$	055.555	-	0,50458	0	-	-	L
$S_0$	055.555	-	0,23411	0	-	-	S
$S_a$	056.554	$h - p_1$	0,01176	0,0410667	0,0000001991	365,2594	S
$S_{sa}$	057.555	$2h$	0,07287	0,0821373	0,0000003982	182,6211	S
$S_{ta}$	058.554	$3h - p_1$	0,00427	0,1232040	0,0000005973	121,7493	S
$M_{sm}$	063.655	$s - 2h + p$	0,01578	0,4715211	0,0000022860	31,8119	L
$M_m$	065.455	$s - p$	0,08254	0,5443747	0,0000026392	27,5546	L
$M_{sf}$	073.555	$2s - 2h$	0,01370	1,0158958	0,0000049252	14,7653	L
$M_f$	075.555	$2s$	0,15642	1,0980331	0,0000053234	13,6608	L
$M_{sm}$	083.655	$3s - 2h + p$	0,00569	1,5695548	0,0000076094	9,5569	L
$M_{tm}$	085.455	$3s - p$	0,02995	1,6424078	0,0000079626	9,1329	L
$M_{sgm}$	093.555	$4s - 2h$	0,00478	2,1139288	0,0000102486	7,0958	L
$2Q_1$	125.755	$\tau - 3s + 2p$	0,00955	12,8442862	0,0000622709	1,1678	S
$\sigma_1$	127.555	$\tau - 3s + 2h$	0,01153	12,9271398	0,0000626725	1,1603	L
$Q_1$	135.655	$\tau - 2s + p$	0,07216	13,3986609	0,0000649585	1,1195	S
$\rho_1$	137.455	$\tau - 2s + 2h + p$	0,01371	13,4715145	0,0000653117	1,1135	L
$O_1$	145.555	$\tau - s$	0,37689	13,9430356	0,0000675977	1,0758	L
$\tau_1$	147.555	$\tau - s + 2h$	0,00491	14,0251729	0,0000679960	1,0695	L
$M_{11}$	155.655	$\tau + p$	0,02964	14,4966939	0,0000702820	1,0347	L
$M_{12}$	155.655	$\tau + p$	0,01040	14,4874103	0,0000702369	1,0295	L
$\chi_1$	157.455	$\tau + 2h - p$	0,00566	14,5695476	0,0000706352	1,0295	L
$\pi_1$	162.556	$\tau + s - 3h + p_1$	0,01029	14,9178647	0,0000723238	1,0055	S
$P_1$	163.555	$\tau + s - 2h$	0,17554	14,9589314	0,0000725229	1,0027	S
$K_1^L$	165.555	$\tau + s$	0,36233	15,0410686	0,0000729212	0,9973	L
$K_1^S$	165.555	$\tau + s$	0,16817	15,0410686	0,0000729212	0,9973	S
$\psi_1$	166.554	$\tau + s + h + p_1$	0,00423	15,0821353	0,0000731203	0,9946	S
$\phi_1$	167.555	$\tau + s + 2h$	0,00756	15,1232059	0,0000733194	0,9919	S
$\theta_1$	173.655	$\tau + 2s - 2h + p$	0,00566	15,5125897	0,0000752072	0,9670	L
$J_1$	175.455	$\tau + 2s - p$	0,02954	15,5854433	0,0000755604	0,9624	L
$SO_1$	183.455	$\tau + 3s - 2h$	0,00492	16,0569644	0,0000778464	0,9342	L
$OO_1$	185.655	$\tau + 3s + N'$	0,01623	16,1391017	0,0000782446	0,9294	L
$\nu_1$	195.455	$\tau + 4s - p$	0,00311	16,6834764	0,0000808838	0,8991	L
$\varepsilon_2$	227.655	$2\tau - 2s + 2p + N'$	0,00671	27,3416964	0,0001325563	0,5486	L
$2N_2$	235.755	$2\tau - 2s + 2p$	0,02301	27,8953548	0,0001352405	0,5363	L
$\mu_2$	237.555	$2\tau - 4s + 4h$	0,02777	27,9682084	0,0001355937	0,5363	L
$N_2$	245.655	$2\tau - s + p$	0,17387	28,4397295	0,0001378797	0,5274	L
$\nu_2$	247.455	$2\tau - s + 2h - p$	0,03303	28,512583	0,0001382329	0,5261	L
$M_2$	255.555	$2\tau$	0,90812	28,9841042	0,0001405189	0,5175	L
$\lambda_2$	263.655	$2\tau - s - 2h + p$	0,00670	29,4556253	0,0001428049	0,5092	L
$L_2$	265.455	$2\tau + s - p$	0,02567	29,5284700	0,0001431580	0,5078	L
$T_2$	272.556	$2\tau + 2s - 3h + p_1$	0,02479	29,5589333	0,0001433058	0,5075	S
$S_2$	273.555	$2\tau + 2s - 2h$	0,42286	30,0000000	0,0001454441	0,5000	S
$R_2$	274.554	$2\tau + 2s - h - p_1$	0,00354	30,0410667	0,0001456432	0,4993	S
$K_2^S$	275.555	$2\tau + 2s$	0,03648	30,0821373	0,0001458423	0,4986	S
$K_2^L$	275.555	$2\tau + 2s$	0,07858	30,0821373	0,0001458423	0,4986	L

Principales composantes extraites du développement de Doodson (source : thèse F. Lefèvre)

# Admittance function

The **admittance function** is defined for each wave as the **ratio between the potential generating tides and the ocean response** :

$$G_n(\varphi, \lambda) = \frac{\xi_n(\varphi, \lambda, t)}{U_n(\varphi, \lambda, t)} \quad \text{with} \quad U_n = gH_n Y_2^{n_1}(\varphi, \lambda) \begin{bmatrix} I \\ -i \end{bmatrix}_{n_1, \text{odd}}^{n_1, \text{even}} e^{i\theta_n}$$

The behavior of the admittance function is smooth in frequency so that it can be interpolated linearly between 2 main waves of close frequencies ( $\dot{\theta}_1$  and  $\dot{\theta}_2$ ) :

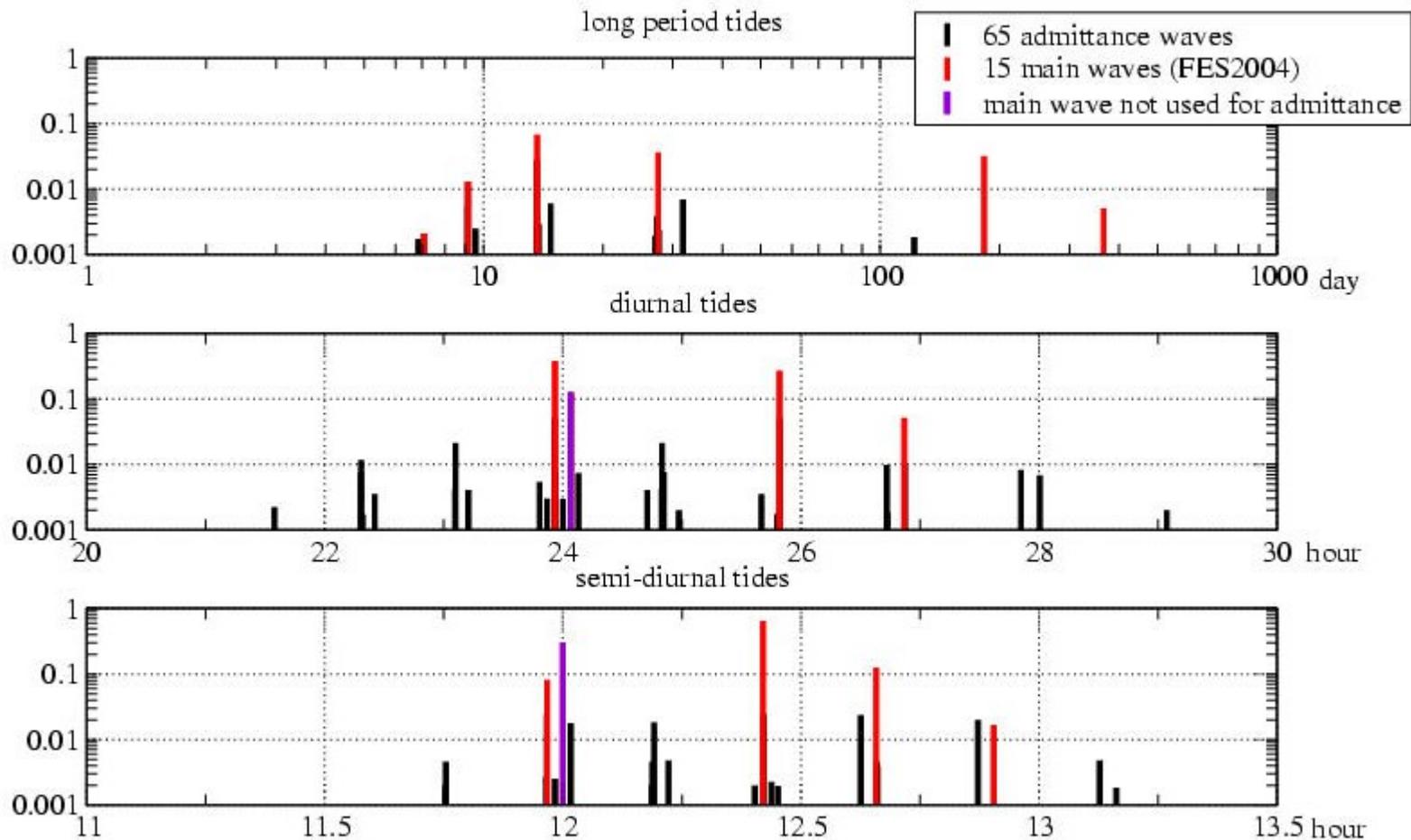
$$G(\dot{\theta}_n) = \frac{\dot{\theta}_n - \dot{\theta}_1}{\dot{\theta}_2 - \dot{\theta}_1} G(\dot{\theta}_2) - \frac{\dot{\theta}_n - \dot{\theta}_2}{\dot{\theta}_2 - \dot{\theta}_1} G(\dot{\theta}_1) \quad \text{with} \quad G(\dot{\theta}_1) = \frac{\xi_1}{U_1} \quad , \quad G(\dot{\theta}_2) = \frac{\xi_2}{U_2}$$

$$\xi_n = G(\dot{\theta}_n) U_n = \frac{\dot{\theta}_n - \dot{\theta}_1}{\dot{\theta}_2 - \dot{\theta}_1} \frac{U_n}{U_2} \xi_2 - \frac{\dot{\theta}_n - \dot{\theta}_2}{\dot{\theta}_2 - \dot{\theta}_1} \frac{U_n}{U_1} \xi_1$$

O. Colombo, thesis of S Casotto, (1989)

$$\begin{aligned} \xi_n = & \frac{\dot{\theta}_n - \dot{\theta}_1}{\dot{\theta}_2 - \dot{\theta}_1} \cdot \frac{H_n}{H_2} \left\{ \cos(\theta_n - \theta_2) \sum_{\ell, m} P_{\ell m}(\sin \varphi) \sum_{+}^{-} [C_{\ell m}^{\pm} \cos(\theta_2 + \chi_2 \pm m\lambda) + S_{\ell m}^{\pm} \sin(\theta_2 + \chi_2 \pm m\lambda)] \right. \\ & \left. + \sin(\theta_n - \theta_2) \sum_{\ell, m} P_{\ell m}(\sin \varphi) \sum_{+}^{-} [S_{\ell m}^{\pm} \cos(\theta_2 + \chi_2 \pm m\lambda) - C_{\ell m}^{\pm} \sin(\theta_2 + \chi_2 \pm m\lambda)] \right\} \\ & + \frac{\dot{\theta}_2 - \dot{\theta}_n}{\dot{\theta}_2 - \dot{\theta}_1} \cdot \frac{H_n}{H_1} \left\{ \cos(\theta_n - \theta_1) \sum_{\ell, m} P_{\ell m}(\sin \varphi) \sum_{+}^{-} [C_{\ell m}^{\pm} \cos(\theta_1 + \chi_1 \pm m\lambda) + S_{\ell m}^{\pm} \sin(\theta_1 + \chi_1 \pm m\lambda)] \right. \\ & \left. + \sin(\theta_n - \theta_1) \sum_{\ell, m} P_{\ell m}(\sin \varphi) \sum_{+}^{-} [S_{\ell m}^{\pm} \cos(\theta_1 + \chi_1 \pm m\lambda) - C_{\ell m}^{\pm} \sin(\theta_1 + \chi_1 \pm m\lambda)] \right\} \end{aligned}$$

# Amplitude of equilibrium tides



Comparison of CHAMP orbits over 10 days  
with the one computed with P1 from FES2004

Orbit differences (mm)	3D	Radial	Normal	Tangential
without P1	144	10	247	36
P1 with admittance	3	1	4	2
admittance (65 waves)	90	16	74	135

**ELLIPSOIDAL CORRECTIONS TO SPHERICAL HARMONICS  
OF SURFACE PHENOMENA GRAVITATIONAL EFFECTS**

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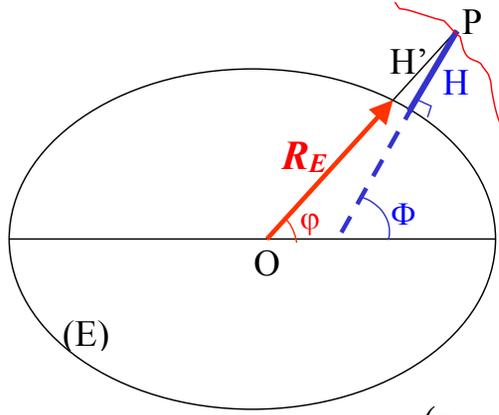
Submitted September 19, 2003

To

Technical University of Graz

Institut für Geodäsie

*Special publication in honour of H. Moritz*



$$H' = H \left(1 + \frac{\eta^2}{2}\right) \quad \text{with} \quad \eta = \Phi - \varphi$$

$$\Phi - \varphi = e^2 \left(1 - \frac{H}{R_0}\right) \sin \Phi \cos \Phi + e^4 \left[1 - \frac{3}{2} \frac{H}{R_0} + \left(\frac{H}{R_0}\right)^2\right] \sin^3 \Phi \cos \Phi$$

$$R_E(\varphi') = R_0(1 - \varepsilon_2 \sin^2 \varphi' + \varepsilon_4 \sin^4 \varphi' + \dots)$$

$$(2 - \delta_{om}) \bar{K}_{lm} = \bar{C}_{lm} - i \bar{S}_{lm}$$

$$\delta \bar{K}_{lm} = \alpha_{lm} \rho \iint_{\sigma_1} \left\{ \int_{R_E(\varphi')}^{r(\varphi', \lambda')} r'^{l+2} dr' \right\} \bar{Y}_{lm}^*(\varphi', \lambda') d\sigma' \quad \text{with} \quad \alpha_{lm} = [(2 - \delta_{om})(2l + 1)MR^l]^{-1}$$

$$\text{with} \quad r' = R_E(\varphi') + H'(\varphi', \lambda') \quad \text{and} \quad R_E(\varphi') = R_0(1 - \varepsilon_2 \sin^2 \varphi' + \varepsilon_4 \sin^4 \varphi' + \dots)$$

$$\left\{ r'^{l+3} / (l + 3) \right\}_{R_E}^{r'} = R_E^{l+3}(\varphi') \left\{ 1 + [H' / R_E(\varphi')] \right. \\ \left. + (l + 2) / 2 [H' / R_E(\varphi')]^2 + \dots - R_E^{l+3}(\varphi') \right\}$$

$$\delta^l \bar{K}_{lm} = \alpha_{lm} \rho \iint_{\sigma_1} \left[ R_0^{l+2} (1 - \varepsilon_2 \sin^2 \varphi' + \varepsilon_4 \sin^4 \varphi')^{l+2} \sum_{n,k} \bar{h}_{nk} \bar{Y}_{nk}(\varphi', \lambda') \right] \cdot \bar{Y}_{lm}^*(\varphi', \lambda') d\sigma'$$

$$\delta^l \bar{K}_{lm}^1 = -\beta_l (l + 2) \varepsilon_2 \left[ a_{l-2,m} \frac{\bar{h}_{l-2,m}}{R_0} + b_{lm} \frac{\bar{h}_{lm}}{R_0} + \bar{c}_{l+2,m} \frac{\bar{h}_{l+2,m}}{R_0} \right]$$

# Ellipsoidal correction

**Model 1:** Ocean tide model: **FES2004** normalized model (in cm)

**Model 2:** Ocean tide model: **FES2004** normalized model **with ellipsoidal correction** (in cm)

				model 1		model 2		2 - 1			
doodson	darw	l	m	c+(cm)	eps+(deg)	c+(cm)	eps+(deg.)	difference	/	relative(%)	
255.555	M2	1	0	0.5888	40.8014	0.5885	39.9910	-0.0003	-0.8104	0.0	-0.5
255.555	M2	2	0	1.2812	180.4369	1.2766	180.0110	-0.0046	-0.4259	-0.4	-0.2
255.555	M2	3	0	1.4460	152.1206	1.4422	152.1420	-0.0038	0.0214	-0.3	0.0
255.555	M2	4	0	1.6966	295.4730	1.6952	295.3580	-0.0014	-0.1150	-0.1	-0.1
255.555	M2	5	0	1.2362	328.6181	1.2222	328.0520	-0.0140	-0.5661	-1.1	-0.3
255.555	M2	1	1	0.3929	140.8622	0.3979	141.2420	0.0050	0.3798	1.3	0.2
255.555	M2	2	1	1.0510	250.4822	1.0551	250.0190	0.0041	-0.4632	0.4	-0.3
255.555	M2	3	1	1.2578	345.4651	1.2555	346.4350	-0.0023	0.9699	-0.2	0.5
255.555	M2	4	1	2.3442	30.4333	2.3297	30.3530	-0.0145	-0.0803	-0.6	0.0
255.555	M2	5	1	3.5506	237.7274	3.5192	237.7810	-0.0314	0.0536	-0.9	0.0
<b>255.555</b>	<b>M2</b>	<b>2</b>	<b>2</b>	<b>5.0115</b>	<b>319.9495</b>	<b>5.0205</b>	<b>319.9080</b>	<b>0.0090</b>	<b>-0.0415</b>	<b>0.2</b>	<b>0.0</b>
255.555	M2	3	2	0.8508	171.8101	0.8549	172.0060	0.0041	0.1959	0.5	0.1
255.555	M2	4	2	4.7345	128.8852	4.7241	129.0010	-0.0104	0.1158	-0.2	0.1
255.555	M2	5	2	1.7639	9.9130	1.7569	9.9690	-0.0070	0.0560	-0.4	0.0
255.555	M2	3	3	3.2388	36.5296	3.2347	36.7060	-0.0041	0.1764	-0.1	0.1
255.555	M2	4	3	3.7611	178.5463	3.7461	178.4950	-0.0150	-0.0513	-0.4	0.0
255.555	M2	5	3	2.8611	295.8639	2.8309	295.7940	-0.0302	-0.0699	-1.1	0.0
255.555	M2	4	4	3.7494	302.6906	3.7439	302.6140	-0.0055	-0.0766	-0.1	0.0
255.555	M2	5	4	4.0702	49.1312	4.0430	49.1210	-0.0272	-0.0102	-0.7	0.0
255.555	M2	5	5	0.8065	70.9801	0.8011	70.6670	-0.0054	-0.3131	-0.7	-0.2

# GRGS-GRACE tide model

Model 1 : Ocean tide model: **FES2004** normalized model (fev. 2004) up to (100,100) in cm  
(long period from FES2002 up to (50,50) + equilibrium Om1/Om2, atmospheric tide

Model 2 : OCEAN TIDES – from **GRACE** solution

doodson	l	m	model 1		model 2		2 - 1		/	relative(%)
			c+(cm)	eps+(deg)	c+(cm)	eps+(deg.)	difference			
135.655 Q1	2	1	0.4240	316.6096	0.4603	313.7090	0.0363	-2.9006	8.6	-1.6
145.555 O1	2	1	2.0262	313.0474	1.9586	315.2550	-0.0676	2.2076	-3.3	1.2
163.555 P1	2	1	0.7478	317.4678	0.6771	316.2990	-0.0707	-1.1688	-9.5	-0.6
165.555 K1	2	1	2.2583	317.3477	2.4783	323.8800	0.2200	6.5323	9.7	3.6
235.755 2N2	2	2	0.1419	341.8343	0.1484	338.2020	0.0065	-3.6323	4.6	-2.0
245.655 N2	2	2	1.1193	328.9101	1.1100	330.6140	-0.0093	1.7039	-0.8	0.9
255.555 M2	2	2	5.0205	319.9080	4.8235	321.1620	-0.1970	1.2540	-3.9	0.7
273.555 S2	2	2	1.8259	315.1121	1.8941	309.9170	0.0682	-5.1951	3.7	-2.9
275.555 K2	2	2	0.4927	316.7372	0.2573	311.0220	-0.2354	-5.7152	-47.8	-3.2

**Mean Annual and Seasonal Atmospheric Tide Models  
based on  
3-hourly and 6-hourly ECMWF Surface Pressure Data**

**In Memory of Peter Schwintzer †**

Richard Biancale

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September 2001 – December 2002

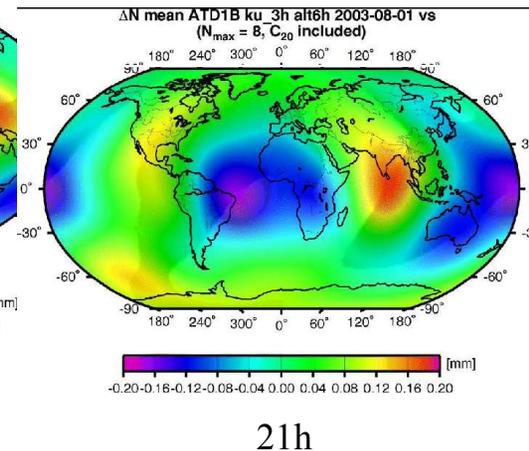
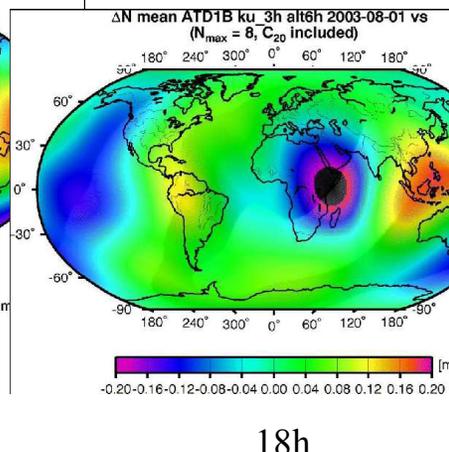
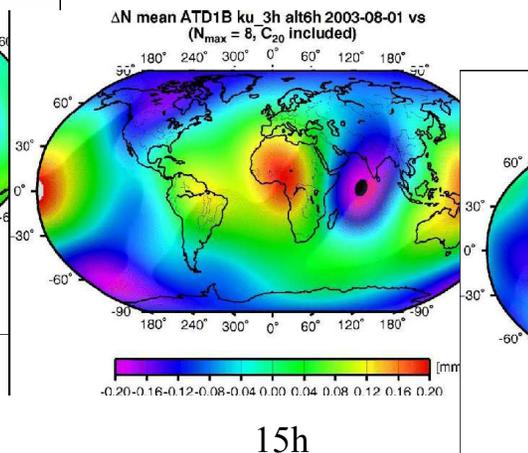
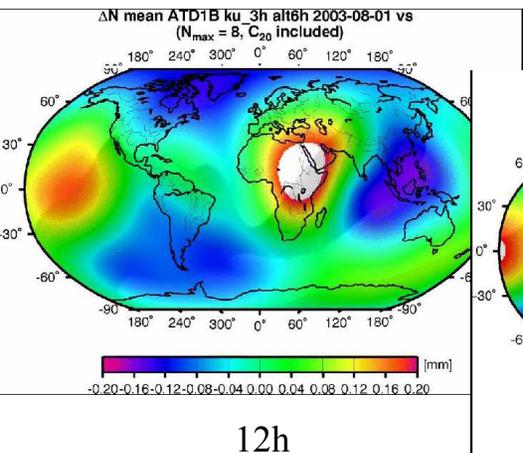
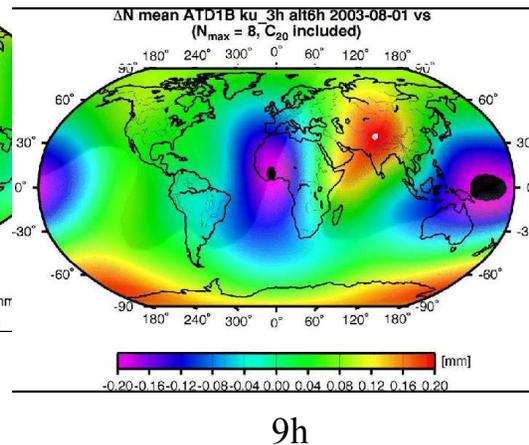
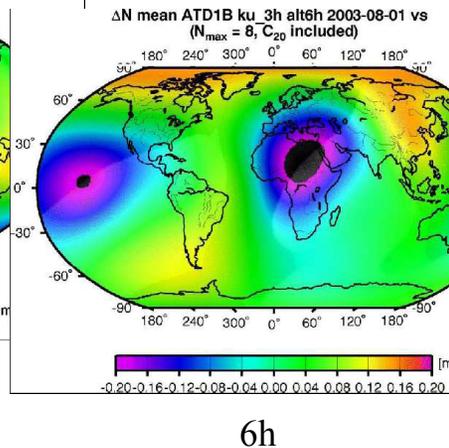
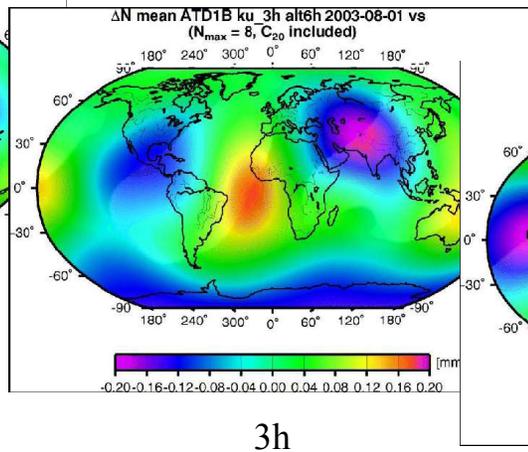
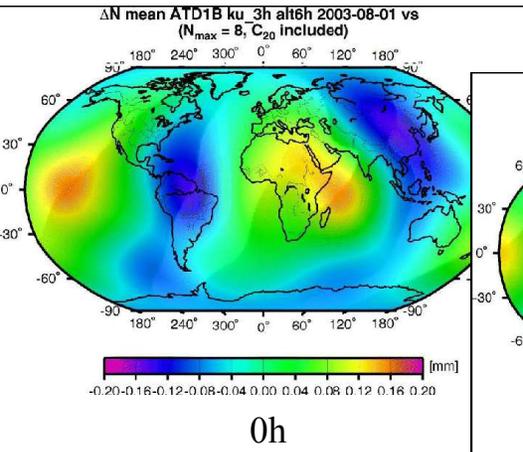
and

Albert Bode

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(retired since May 1, 2002)

# Temporal evolution of the geoid differences derived from the mean atmospheric tide models

*based on 6h ECMWF pressure field data (1985-2002) for August 2, each 3h.*



## S1

## S2

ECMWF derived atmospheric tides (Bode & Biancale,2003)  
normalized model (in mbar)

Doodson	Darw	l	m	Csin+	Ccos+	C+	eps+
164.556	S1	0	0	-0.010427	0.004887	0.0115	295.112
164.556	S1	1	0	-0.011572	0.007151	0.0136	301.714
164.556	S1	2	0	-0.021999	-0.019367	0.0293	228.641
164.556	S1	3	0	-0.019447	-0.001090	0.0195	266.792
<b>164.556</b>	<b>S1</b>	<b>1</b>	<b>1</b>	<b>0.211548</b>	<b>0.063876</b>	<b>0.2210</b>	<b>73.199</b>
164.556	S1	2	1	0.011431	0.014826	0.0187	37.633
<b>164.556</b>	<b>S1</b>	<b>3</b>	<b>1</b>	<b>-0.125362</b>	<b>-0.011301</b>	<b>0.1259</b>	<b>264.849</b>
164.556	S1	4	1	0.006730	-0.015007	0.0164	155.846
164.556	S1	2	2	0.040451	0.052284	0.0661	37.728
164.556	S1	3	2	-0.000839	0.024861	0.0249	358.067
164.556	S1	4	2	-0.027001	-0.014535	0.0307	241.706
164.556	S1	5	2	0.001710	-0.002672	0.0032	147.382
164.556	S1	3	3	-0.016213	-0.011110	0.0197	235.579
164.556	S1	4	3	-0.028287	0.015258	0.0321	298.342
164.556	S1	5	3	-0.018513	-0.003835	0.0189	258.297
164.556	S1	6	3	-0.001229	-0.011130	0.0112	186.301
164.556	S1	4	4	-0.011813	0.026961	0.0294	336.339
164.556	S1	5	4	0.013651	-0.002437	0.0139	100.122
164.556	S1	6	4	0.005458	0.007862	0.0096	34.769
164.556	S1	7	4	-0.002848	0.010407	0.0108	344.695
164.556	S1	5	5	-0.019909	0.031089	0.0369	327.365
164.556	S1	6	5	0.034243	-0.001804	0.0343	93.016
164.556	S1	7	5	0.011923	0.001275	0.0120	83.896
164.556	S1	8	5	-0.006841	-0.000462	0.0069	266.136

ECMWF derived atmospheric tides (Bode & Biancale,2003)  
normalized model (in mbar)

Doodson	Darw	l	m	Csin+	Ccos+	C+	eps+
273.555	S2	0	0	-0.001267	0.004729	0.0049	345.001
273.555	S2	1	0	0.003017	-0.001218	0.0033	111.985
273.555	S2	2	0	0.046910	-0.110495	0.1200	156.997
273.555	S2	3	0	0.011240	0.003652	0.0118	72.000
273.555	S2	1	1	0.025133	-0.003030	0.0253	96.874
273.555	S2	2	1	0.011672	-0.022752	0.0256	152.842
273.555	S2	3	1	-0.019507	0.009509	0.0217	295.988
273.555	S2	4	1	-0.004188	0.001154	0.0043	285.406
<b>273.555</b>	<b>S2</b>	<b>2</b>	<b>2</b>	<b>0.283369</b>	<b>-0.468298</b>	<b>0.5474</b>	<b>148.822</b>
273.555	S2	3	2	0.009812	0.026639	0.0284	20.220
<b>273.555</b>	<b>S2</b>	<b>4</b>	<b>2</b>	<b>-0.036313</b>	<b>0.070506</b>	<b>0.0793</b>	<b>332.750</b>
273.555	S2	5	2	-0.004402	-0.002872	0.0053	236.878
273.555	S2	3	3	0.016795	0.005516	0.0177	71.818
273.555	S2	4	3	0.003813	-0.000086	0.0038	91.292
273.555	S2	5	3	-0.004878	0.006430	0.0081	322.815
273.555	S2	6	3	0.001879	0.001580	0.0025	49.940
273.555	S2	4	4	-0.015098	0.000828	0.0151	273.139
273.555	S2	5	4	0.002428	0.008129	0.0085	16.630
273.555	S2	6	4	-0.001503	0.000413	0.0016	285.365
273.555	S2	7	4	-0.004306	-0.001009	0.0044	256.812
273.555	S2	5	5	0.009324	-0.000794	0.0094	94.867
273.555	S2	6	5	0.000304	0.000524	0.0006	30.120
273.555	S2	7	5	0.003957	-0.000054	0.0040	90.782
273.555	S2	8	5	0.005283	0.001430	0.0055	74.854

## Sa

## Ssa

ECMWF derived atmospheric tides (Bode & Biancale,2003)  
normalized model (in mbar)

Doodson	Darw	l	m	Csin+	Ccos+	C+	eps+
56.554	Sa	0	0	-0.168856	-0.049683	0.1760	253.604
56.554	Sa	1	0	0.105071	-0.057879	0.1200	118.848
56.554	Sa	2	0	0.071952	0.653729	0.6577	6.281
56.554	Sa	3	0	-1.225490	-0.371665	1.2806	253.129
56.554	Sa	1	1	0.047282	0.220890	0.2259	12.082
56.554	Sa	2	1	0.306172	0.548859	0.6285	29.154
56.554	Sa	3	1	-0.042330	0.336106	0.3388	352.822
56.554	Sa	4	1	-0.112705	-0.170263	0.2042	213.502
56.554	Sa	2	2	-0.218981	0.158912	0.2706	305.968
56.554	Sa	3	2	-0.256851	-0.042370	0.2603	260.633
56.554	Sa	4	2	-0.264284	0.003188	0.2643	270.691
56.554	Sa	5	2	-0.197183	-0.006217	0.1973	268.194
56.554	Sa	3	3	0.133799	-0.031722	0.1375	103.338
56.554	Sa	4	3	0.096020	0.241783	0.2602	21.660
56.554	Sa	5	3	0.181815	0.111700	0.2134	58.435
56.554	Sa	6	3	0.017368	0.156099	0.1571	6.349
56.554	Sa	4	4	0.013902	-0.131590	0.1323	173.969
56.554	Sa	5	4	-0.103664	-0.034022	0.1091	251.830
56.554	Sa	6	4	0.061234	-0.034274	0.0702	119.237
56.554	Sa	7	4	-0.053530	0.014686	0.0555	285.342
56.554	Sa	5	5	-0.013892	-0.035348	0.0380	201.455
56.554	Sa	6	5	0.093559	-0.112805	0.1466	140.328
56.554	Sa	7	5	0.056152	-0.089431	0.1056	147.876
56.554	Sa	8	5	0.047282	-0.034263	0.0584	125.929

ECMWF derived atmospheric tides (Bode & Biancale,2003)  
normalized model (in mbar)

Doodson	Darw	l	m	Csin+	Ccos+	C+	eps+
57.555	Ssa	0	0	0.026418	-0.043967	0.0513	149.000
57.555	Ssa	1	0	0.213356	0.369254	0.4265	30.019
57.555	Ssa	2	0	-0.037297	0.093489	0.1007	338.251
57.555	Ssa	3	0	0.440373	0.198389	0.4830	65.748
57.555	Ssa	1	1	-0.187842	-0.054635	0.1956	253.783
57.555	Ssa	2	1	-0.100279	0.028146	0.1042	285.678
57.555	Ssa	3	1	0.022220	0.062821	0.0666	19.479
57.555	Ssa	4	1	0.025233	0.041395	0.0485	31.365
57.555	Ssa	2	2	-0.033028	0.012868	0.0354	291.286
57.555	Ssa	3	2	-0.071309	0.002881	0.0714	272.314
57.555	Ssa	4	2	0.028970	0.007771	0.0300	74.984
57.555	Ssa	5	2	-0.028689	-0.015389	0.0326	241.791
57.555	Ssa	3	3	-0.075016	-0.066689	0.1004	228.363
57.555	Ssa	4	3	0.007825	-0.056142	0.0567	172.065
57.555	Ssa	5	3	-0.111399	-0.068246	0.1306	238.507
57.555	Ssa	6	3	-0.022692	-0.077537	0.0808	196.313
57.555	Ssa	4	4	-0.003236	-0.013119	0.0135	193.856
57.555	Ssa	5	4	0.048517	-0.037387	0.0613	127.618
57.555	Ssa	6	4	-0.013902	0.014806	0.0203	316.804
57.555	Ssa	7	4	0.035921	0.013320	0.0383	69.655
57.555	Ssa	5	5	0.004073	-0.025976	0.0263	171.089
57.555	Ssa	6	5	0.000599	-0.026941	0.0269	178.726
57.555	Ssa	7	5	-0.037086	-0.009590	0.0383	255.502
57.555	Ssa	8	5	-0.028427	-0.007664	0.0294	254.912

# Comparison to the Haurwitz and Cowley model

## Model 1 : H & W atmospheric tides (Haurwitz & Cowley, 1973)

normalized model up to (8,5) (in mbar)

## Model 2 : ECMWF derived atmospheric tides (Bode & Biancale, 2003)

normalized model up to (8,5) (in mbar)

doodson	model 1		model 2		2 - 1		/	relative(%)		
	l	m	c+(mb)	eps+(deg)	c+(mb)	eps+(deg.)		difference		
<b>164.556 S1</b>	<b>1</b>	<b>1</b>	<b>0.2885</b>	<b>12.0001</b>	<b>0.2210</b>	<b>73.1990</b>	<b>-0.0675</b>	<b>61.1989</b>	<b>-23.4</b>	<b>34.0</b>
164.556 S1	2	1	0.0365	330.9997	0.0187	37.6330	-0.0178	66.6333	-48.8	37.0
<b>164.556 S1</b>	<b>3</b>	<b>1</b>	<b>0.0860</b>	<b>197.0001</b>	<b>0.1259</b>	<b>264.8490</b>	<b>0.0399</b>	<b>67.8489</b>	<b>46.4</b>	<b>37.7</b>
<b>273.555 S2</b>	<b>2</b>	<b>2</b>	<b>0.5510</b>	<b>159.0000</b>	<b>0.5474</b>	<b>148.8220</b>	<b>-0.0036</b>	<b>-10.1780</b>	<b>-0.7</b>	<b>-5.7</b>
273.555 S2	3	2	0.0260	81.0007	0.0284	20.2200	0.0024	-60.7807	9.2	-33.8
<b>273.555 S2</b>	<b>4</b>	<b>2</b>	<b>0.0540</b>	<b>330.9995</b>	<b>0.0793</b>	<b>332.7500</b>	<b>0.0253</b>	<b>1.7505</b>	<b>46.9</b>	<b>1.0</b>